Figured Bass and Tonality Recognition

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ABSTRACT

In the course of the WedelMusic project [15], we are currently implementing retrieval engines based on musical content automatically extracted from a musical score. By musical content, we mean not only main melodic motives, but also harmony, or tonality.

In this paper, we first review previous research in the domain of harmonic analysis of tonal music.

We then present a method for automated harmonic analysis of a music score based on the extraction of a figured bass. The figured bass is determined by means of a template-matching algorithm, where templates for chords can be entirely and easily redefined by the end-user. We also address the problem of tonality recognition with a simple algorithm based on the figured bass.

Limitations of the method are discussed. Results are shown and compared to previous research.

Finally, potential uses for Music Information Retrieval are discussed.

KEYWORDS

Music analysis, automatic extraction of musical features, figured bass and tonality recognition.

1. INTRODUCTION

As stated by Ian Bent in his article "Analysis" of the New Grove's Dictionary, musical analysis is "the resolution of a musical structure into relatively simpler constituent elements, and the investigation of the functions of these elements within that structure".

Harmonic analysis is one of the principal means to achieve this goal through the production of a figured bass and the analysis of the function of chords based on the relationship of their root to the main tonality. In this paper, we describe a technique for the automated extraction of the figured bass.

The figured bass is a very old principle, described in several treatises, starting from "Del sonare sopra il basso" by Agazzari (1607).

The aim of the figured bass was, in principle, oriented towards interpretation. Rameau turned it into a genuine theory of tonality with the introduction of the fundamental concept of root. Successive refinements of the theory have been introduced in the 18th, 19th (e.g., by Reicha and Fetis) and 20th (e.g., Schoenberg [10, 11]) centuries. For a general history of the theory of harmony, one can refer to Ian Bent [1] or Jacques Chailley [2]

Several processes can be build on the top of a harmonic reduction

- detection of tonality,
- recognition of cadence,
- detection of similar structures

Following a brief review of systems addressing the problem of tonal and harmonic analysis, we first point out the problems raised by harmonic reduction. We then describe our algorithm, and show its use in some examples. In the subsequent section, we show the application of a simple process of tonality detection on top of harmonic reduction.

The analysis tools that are described here are part of the WedelMusic project, which is funded by the European Commission [15]. Its aim is the development of a system of distribution of music scores over the Internet while preserving the owner's rights. This project includes a cataloguing system. Indexes are built from such metadata as name of composer, date of composition and so on. Indexes are also built on the basis of musical content, as extracted from the score by analysis tools developed at Ircam. They include such elements as main motives, descriptions of tonalities and their relation with the main tonality, etc.

These elements can be used in a more general strategy of Music Information Retrieval, which would be based not only just on motives, but also on tonal style, harmony and so on.

2. A BRIEF TOUR OF MUSIC ANALYSIS SYSTEMS

In the past, a number of systems have been developed to address the problem of automatic tonal harmonic analysis. Only a few tackle the difficult problem of chord generation - that is, generation of root and encoding of the nature of the chord - directly from the score.

Maxwell's expert system for harmonic analysis of tonal music [6] is a rule-based system, consisting of more than 50 rules. The first phase performs a reduction of the vertical sonorities of the piece into a chord sequence, by recognizing dissonances and consonances. Maxwell's complex set of decision rules for consonance and dissonance is difficult to adapt to situations where the notion of dissonance is slightly different, such as music of the 19th century. In addition, as noticed by David Temperley [12], Maxwell's algorithm appears not to be capable of correctly handling situations where notes of the chord are stated in sequence.

Temperley's approach to harmonic analysis [12] consists of a set of preference rules, as described in Lerdahl's and Jackendoff's generative theory of tonal music [5]. As in Maxwell's system, the first phase of Temperley's algorithm leads to the production of the roots of chords. Despite the strongly encouraging results he achieved, the author himself pointed out several problems with the algorithm, especially in the analysis of the Gavotte from the French Suite n° 5 by J.-S. Bach.

Pardo and Birmingham [8] developed HarmAn, a system that partitions tonal music into harmonically significant segments corresponding to single chords. It also tags these segments with the proper chord label. A strength of the system is that it is independent of rhythm. New templates for chords can be introduced, but this requires a rethinking of both the preferences rules and the scoring method for a single template, as stated by the authors. A numerical method is used for scoring elements, with known drawbacks: as stated by Francois Pachet [7], "numerical values are difficult to justify, difficult to maintain, and have poor explanatory capacity". The system works with a MIDI-like representation of notes, and no enharmonic spelling algorithm is implemented. The system thus suffers from a number of drawbacks by not recognizing the difference between, for example, F# and Gb. This will certainly lead to a number of problems in passages belonging to tonalities with several accidentals. In addition, some aggregations used in the late 18th century and in the 19th century, such as the augmented sixth (C – E – G – A#) cannot be distinguished from other chords (in this case, from a seventh on the fifth degree).

Other systems have been developed, which don't address the first difficulty of chord recognition and segmentation of the score.

Winograd [14], in a pioneering work, addressed the analysis of musical scores by using systemic grammars. His method needs a preliminary hand-made conversion of the original score into a score expressed as a sequence of four-part perfect chords. During this operation, ornamental notes, like suspensions, passing notes and the like, are eliminated.

Ulrich [13] developed a process of functional analysis, this term referring to the identification of the function of each chord in a song, and the grouping together of measures that move the tune from one key center to another one. Similarly to Winograd, the input to the program consists of a sequence of chords, each of them consisting of a set of musical notes. An interesting part of the system is an algorithm for detection of keys, described as an "island-growing" mechanism.

François Pachet's approach to computer analysis of jazz chord sequences [7] can be seen as an extension of Ulrich's island growing mechanism, as stated by the author himself. The input of the system is a chord sequence, already explicitly mentioned on the score. The most important improvement to Ulrich's mechanism is that the system outputs a hierarchical description of modulations.

Hoffmann and Birmingham [4] use a constraint satisfaction approach in order to solve the problem of tonal analysis. Similarly to Winograd's method, a preliminary hand-made conversion of the score is necessary.

3. PROBLEM STATEMENT

The issues raised by harmonic reduction are the following:

- ornamental notes and incomplete harmony,
- ambiguities,
- non-regularity of harmony,
- non-universality of harmony rules.

We shall now address each of these points.

3.1 Ornamental notes and incomplete harmonies

In the process of harmonic reduction, some notes are extraneous to the harmony - these are ornamental notes, like appoggiaturas, suspensions, passing notes and so on. On the other hand, harmony is frequently incomplete – i.e., some notes may be missing.

This is illustrated in the following example:

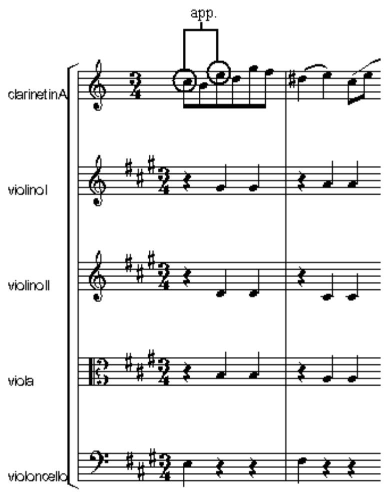

Figure 1. Ornamental notes and incomplete harmonies (Trio of the Clarinet Quintet by Mozart, KV 581)

The circled A and C# (written transposed C and E) in the clarinet part in the first measure are not part of the harmony, thus they are to be considered as ornamental notes.

In the second measure, the harmony – a fifth chord on F# - is never complete anywhere in the measure.

To cope with these problems, we must apply a fundamental rule of analysis, as described by Cook [3] in his treatise: "Essentially there are two analytical acts: the act of omission and the act of relation". In order to decide if a note is an ornamental, we use the rule handling the resolution: in general, resolution of an ornamental note such as a suspension, a passing note, an appoggiatura is performed with a conjunct degree.

In some cases, however, the resolution of an ornamental will be done through disjoint motion: for example, a suspension can be resolved by first playing a harmonic note before playing the resolution. For now, we only apply “natural” resolution, and we will extend our rule to handle more cases.

Another rule for deciding if a note is an ornamental is based on the relative duration (weight) of the note as compared to the other notes of the chord.

3.2 Ambiguities

Some ambiguities have to be resolved, since certain vertical aggregations are not “true harmony”, as shown in the following example:

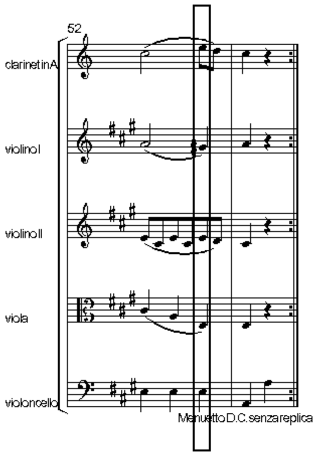

Figure 2. Ambiguous harmony.
(Trio of the Clarinet Quintet by Mozart, KV 581)

The harmony found on the third beat, surrounded here by a rectangle, looks like a sixth. If it is so analysed, its root would then be C#, the third degree of A Major. But this is a nonsense in this context.

3.3 Non-regularity of harmony

In traditional harmony, one cannot assume that the harmonic rhythm is regular. In other words, a harmonic reduction process cannot be based on the assumption that harmony is the same for a beat and for a measure.

3.4 Non-universality of harmony rules

The “theory of harmony” is not to be considered as a genuine, universal and well-defined set of rules. As François Pachet states [7], it is “a theory without theorems”. Rules of harmony have evolved through history. As noticed by Hoffmann [4], “the rules for tonal harmony are not specifically stated, but are conventions drawn from centuries of musical experience”.

To cope with this problem, we must let the user define his own sets of “harmonic rules”, and choose which set of “right” rules to apply.

4. DESCRIPTION OF HARMONIC REDUCTION

The harmonic reduction process includes two main phases:

- a *clusterisation*, which is composed of a first phase of *quantization* of each measure, followed by a vertical aggregation of notes belonging to the same quantization,
- an iterative horizontal aggregation, in which unnecessary notes are eliminated, and successive clusters are merged into chords.

The *quantization* of the measure is simply the computation of the duration of the shortest note in the measure.

For each quantization, we store the result of a vertical aggregation as a *cluster*. We use this term here to designate an aggregate of notes which has not yet reached the status of a chord; it is represented as a list of notes, each of them stored with its diatonic value (pitch, accidental, octave) and its melodic function (interval to the following note in the same voice). The information about the melodic function is used to decide whether the note is an ornamental note or a harmonic note: a note with an interval of a second to the following note is considered to be a possible ornamental note. A note with an interval greater than a second (a third, a fourth and so on) is considered to be a harmonic note.

The iterative horizontal aggregation uses a set of user-defined chord templates, i.e., a list of chords together with their figures. For the analysis presented in this paper, we have used a set of 33 chords, including some seventh and ninth chords, which can be considered as representative of classical harmony as used by composers at the end of the 18th century.

For other styles, the user can choose another pre-defined set of chords, or to redefine entirely his own set of chords, and store it in the database. The definition of the set of chords is easily input through the Wedel score editor, which is also used for displaying the score being analysed. The process of horizontal aggregation extensively uses the set of chords that the user has selected.

We begin the process of aggregation by comparing two consecutive clusters. They are considered the same if the sounds composing the two clusters are the same, regardless of their octave, i.e., each sound of the first cluster belongs to the second, and each sound of the second belongs to the first. In this case, the two clusters are merged in one.

If they have not been merged, the process performs a union of both clusters and compares the result against the each chord in the set of chords:

- If the union, except for the possible ornamental notes, can be exactly mapped to a chord, the two clusters are merged into one.
- If the union, including the possible ornamental notes, can be exactly mapped to a chord, the two clusters are merged into one, and the ornamental notes are now considered to be harmonic notes.

The merge is first applied beat by beat, and then measure by measure, and is iteratively repeated until no more merge can be achieved.

When no further merge can be accomplished, an attempt is made to turn each cluster into a chord, by mapping it to the nearest chord possible.

First, we try to find a chord containing all the harmonic notes of the cluster and conversely. If this attempt fails, we then search for a chord containing all the notes of the cluster (this assumes that the cluster can be an incomplete chord). If this fails, we try to find a chord such that the duration of those cluster's notes which cannot be mapped to any note of this chord, is significantly shorter (actually by a factor of 6) than the total duration of the notes of the cluster (this assumes that these notes are really ornamental notes, but were not previously detected as being so).

5. EVALUATION

5.1 Limitations

Some very special cases are not taken into account in our algorithm, notably pedals. Another limitation is due to the oversimplicity of our rule for detection of ornamental notes: some ornamentals can be followed by a disjoint interval, and these can only be detected by the last attempt of turning a cluster into a chord, as described above.

A further limitation is due to the fact that our algorithm doesn't take sufficiently into account the context. Some problems of context dependencies are handled, as shown below in fig. 5, but the resolution of ambiguities is not sufficiently strong. Let us examine this example extracted from the Gavotte from the French Suite n° 5 by J.-S. Bach:

Figure 3. Gavotte from French Suite n°5 by J.-S. Bach, measure 8

In this Gavotte, whose figured bass is given below (see Figure 11), the harmony is a seventh chord on the dominant of D (A - C# - E - G). But in some other contexts, it can be a sixth chord on the root of F# (this analysis being the one produced by our algorithm).

More generally, we must limit the scope of our harmonic analysis to accompanied melody, even if in some limit cases of monophonic voice, a good result can be obtained (as shown below with Mozart's example). We think also that these results can be applied to some music of the 20th century, for example Bartok's works, by redefining the set of chords, but we are aware that this method cannot be applied to contrapuntal work.

5.2 Examples

These examples show the process of harmonic reduction applied to the Trio of Mozart's Clarinet Quintet.

The first example[1] shows elimination of ornamental notes and reconstruction of incomplete chords

Figure 4. Elimination of ornamental notes and reconstruction of incomplete harmony. (Trio of the Clarinet Quintet by Mozart, KV 581)

The figure 5 shows the resolution of ambiguities:

Figure 5. Resolution of ambiguities (Trio of the Clarinet Quintet by Mozart, KV 581)

The harmony on the third beat is not analysed as being a 6th chord, as the C# in the clarinet part is determined as a potential ornamental note (an appoggiatura), and thus, the harmony is merged with the following one, giving as a result a correct analysis of a 7th chord on the fifth degree.

The following example shows that the algorithm can produce correct results even in the case of a simple monophonic voice:

[1] The notation of figures follows the conventions of figured bass as stated in the treatise, with the following exceptions: figures are written from left to right and not from top to bottom, and a slash following a figure indicates that this figure is diminished.

7+ is for $\overset{7}{+}$, 65/ is for $\overset{6}{5\!\!/}$.

Figure 6. Detection of the root for a monophonic voice (Trio of the Clarinet Quintet by Mozart, KV 581)

The root is correctly detected as being a B.

This last example shows that detection of figured bass is not constrained by rhythm:

Figure 7. Measures 6 – 7 , Sarabande in D minor by J.-S. Bach

6. Application to tonality detection

On top of this harmonic reduction, we have developed a simple algorithm of tonality detection. This algorithm is based on the fact that each chord can belong to a limited number of tonalities.

The possible tonalities are derived from the figured bass as previously obtained, and a process of elimination is then applied by successively merging regions where there is at least one tonality in common, eliminating tonalities not common to the regions being merged. Where there is no common tonality, a change of tonality is therefore detected.

This algorithm, proceeding as an "island-growing" mechanism, is very near to the system implemented by Ulrich.

The result of this operation for the Trio of the Clarinet Quintet by Mozart is shown here, together with the complete figured bass generated by the system:

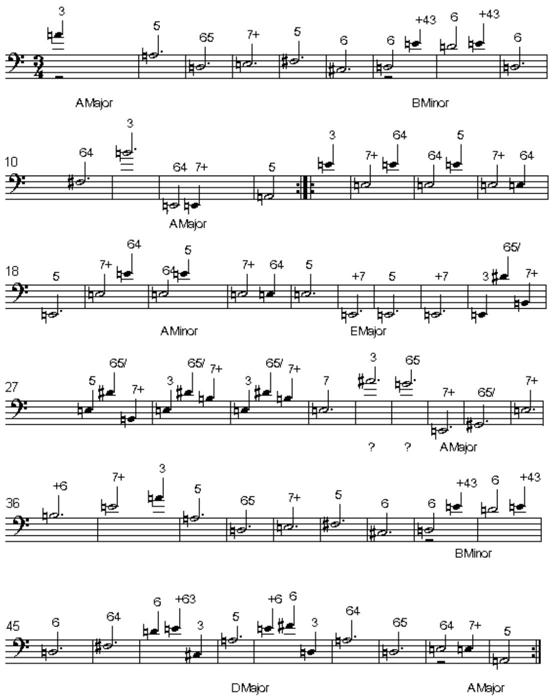

Figure 8: Figured Bass and Tonalities detected for the Trio of the Clarinet Quintet by Mozart, KV 581

The figured bass presented here is totally consistent with an analysis done by a human analyst, with a small exception (in measures 31 and 32).

The detected tonalities are written below the figured bass. When a change of tonality is detected, it is written on the score, the tonality is determined to be the same until the next change of tonality. If a tonality is not recognized, it is denoted bys "?".

The tonalities are correctly detected as being A Major, B Minor, A Minor, E Major and D Major, with the exception of measures 31 and 32 where the tonality is unrecognised.

The advantage of this approach is that, due to the harmonic reduction process, a number of problems related to tonality recognition are easily solved.

In particular, certain notes "out of the tonality", that is, notes which are not really part of the tonality, are eliminated from the process. One can notice, using the original score, that a B# in measure 5 or a E# in measure 51 are completely ignored and do not interfere with the process of tonality recognition.

However, some problems are raised by this simplification.

In the following example from "Eine Kleine Nachtmusik" by Mozart, measures 24-28, a main tonality is simply ignored:

Figure 9. Mozart's "Eine Kleine Nachtmusik"

The musicologist easily recognizes in measure 28 the main entry of the second theme, in D Major.

Unfortunately, the G natural is ignored by the process of harmonic reduction, being a passing note, even if the root harmony is correctly recognized as D. So, between the (short) modulation in A found at the end of measure 25, and the (short) modulation in E minor correctly recognized at the end of measure 28, the main tonality of D Major is not recognized.

A possible solution to this problem can be a refinement of the model of tonality recognition by adding a rule recognizing some modulations as being embedded modulations (in some French treatises of Harmony, such modulations are called "emprunts", i.e., "loans"). To this end, a derivation of the model of François Pachet can be applied.

7. COMPARISON

For the purpose of comparing our models with other work, we show here the result of the production of figured bass applied to a fragment of a Sarabande in D minor by J.-S. Bach, whose analysis can be found in the papers of Maxwell [6] and Pardo [8]:

Figure 10. Sarabande in D minor by J.-S. Bach

The result of the production of figured bass is shown here on the third staff, marked "FB", together with the recognized tonalities.

The results of Pardo and Maxwell are shown on the following lines.

The results of Maxwell are identical to ours, with a (very little) exception at the beginning of measure 5: the reason is that chord Bb – D - F# - A is part of our templates. In measure 8, Maxwell's system doesn't recognize the sixth-fourth chord on the root of D.

Pardo's result suffers from several drawbacks: the system produces an A Major chord on the second eighth note of measure 2, and a G Major chord on the second eighth note of measure 4, this last one being quite annoying since the correct tonality in this context is G Minor. Incorrect analysis of augmented chords on the first beat of measure 5 and on the third beat of measure 6 are certainly due to the MIDI-like representation of notes. In addition, one cannot understand the analysis of the last chord (A7), the seventh - G - being not in the chord.

We have also applied the Figured Bass to the Gavotte already analysed by Temperley [12].

Figure 11. Gavotte from French Suite n°5 by J.-S. Bach

There are several drawbacks in this Figured Bass:

- measure 6 is incorrectly analysed as a seventh of dominant on the tonic, this chord being part of our templates,
- the last chord of measure 8 is incorrectly analysed as a sixth and fifth chord,
- the root of measure 5 is correctly detected, but incorrectly figured as a seventh chord.

Temperley's analysis of the same Gavotte also suffers from several drawbacks:

- measure 8 is incorrectly analysed as entirely based on the root of D,
- a incorrect root of A is detected for the second beat of measure 4,
- the second half of measure 5 is incorrectly detected as being based on the root of E,
- incorrect roots of D and E are detected in measure 6.

Temperley's analysis of measure 6 can be considered better than our Figured Bass, but our analysis of measure 4 can be considered better. The definite mistake made in both cases in measure 8 is due to the same fact: our models are not able to analyse correctly the last F# as an ornamental.

8. Conclusion and perspectives

In this paper, we described an algorithm for production of a figured bass.

This algorithm allows the musicologist to redefine "harmony rules" entirely, merely by redefining the chord templates. It is thus much more general than algorithms found in the literature. We have also shown that our results can be considered at least as good as the best results previously found. We are currently trying to make improvements to the algorithm.

We have shown that higher-level processes, for example tonality recognition, can be build on the top of the figured bass. As stated in the introduction, several processes can be build on the top of a figured bass: detection of cadence, of tonality, of similar structures, and so on. Results of these processes can be stored and indexed in database for the purpose of Music Information Retrieval.

One can notice that the Figured Bass, as the result of a standardized process, can be used as a retrieval criterion. It is a useful criterion for a teacher, for example, in the retrieval of scores using of the same fragment of Figured Bass (the Figured Bass of Sarabande in D minor by J.-S. Bach is an interesting one). To this end, a transposition independent encoding of the Figured Bass must be developed, and we are currently working on it.

Other applications for Music Information Retrieval are possible, such as classification of style based upon the frequency of chords, or upon the relationship between the recognized tonalities and the main tonality, assuming that the complexity of tonal relations is characteristic of a given style. Some techniques actually used to classify melody, such as the Hidden Markov Model (Pollastri, [9]), or techniques issued from Graph Theory can be also applied on the description of the score generated by the harmonic analysis.

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